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MULTIPLE LIMIT CYCLES FOR CHEMICAL OSCILLATOR

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Abstract. We show that there exist multiple limit cycles for a model of chemical oscillation which is proposed by Lengyel et al. [4]. Using the modified formula to compute the coefficients of the normal form, we derive the exact parameter values such that the equilibrium is a weak focus of order two. Then, adding some suitable perturbations to parameters, we construct a concrete example which has two limit cycles.

Keywords. chemical oscillation, multiple limit cycles

1 Introduction

In this paper, we consider the existence of multiple limit cycles for a model of the chemical oscillation in the chlorine dioxide–iodine–malonic acid reaction. We deal with the system

$$\begin{cases} x' = a - x - \dfrac{4xy}{1+x^2} \\ y' = b\,x\Big(1 - \dfrac{y}{1+x^2}\Big) \end{cases} \tag{1.1}$$

where x and y represent the concentrations of I^- and ClO_2^- respectively. System (1.1) is proposed and analyzed by Lengel et al. [4], and it is shown that there exists a stable limit cycle under certain parameter values by the Poincare–Bendixon Theorem. System (1.1) is also introduced in several elementary textbooks [2, 7] as an example which has a limit cycle. Especially, Strogatz explains the existence of the limit cycle is due to a supercritical Hopf bifurcation. However, this explanation is incomplete because system (1.1) can have an unstable limit cycle by a subcritical Hopf bifurcation. Moreover, there can exist multiple limit cycles under certain parameter values.

It is well known that, if the system has a weak focus of order k, then we can generate k limit cycles with suitable perturbations [6]. In order to derive the condition that the equilibrium is a weak focus, we may have to convert the system into the normal form. Although several methods have been proposed [1, 3], it is not easy to compute the coefficients of the normal form because it requires long tedious calculations. To compute the coefficients