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NONLINEAR SMALL-GAIN THEOREMS FOR LARGE-SCALE TIME-DELAY SYSTEMS

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Abstract. This paper presents nonlinear small-gain theorems for input-to-state stability and input-to-output stability properties of large-scale time-delay systems consisting of multiple subsystems based on Razumikhin-type techniques. A key strategy is to follow the spirit of the Razumikhin theorem and treat the state variables with delays as disturbances to the system in question. A consequence of this approach is that the problem of stability analysis for large-scale systems described by functional differential equations is converted into a more tractable one for state-delay-free systems with disturbances.

Keywords. time-delay systems, input-to-state stability, input-to-output stability, nonlinear small-gain, Razumikhin theorems.

AMS (MOS) subject classification: 93D05, 93D09, 34K20

1 Introduction

Time delays are a common occurrence in various applications, including physical and chemical processes, engineering systems, and biological systems. It is thus critical to develop tools for stability analysis for systems with time delays. One frequently used approach for systems with delays is via the construction of Lyapunov-Krasovskii functionals. While this method can be carried out without specific knowledge of trajectories, the task of constructing a Lyapunov-Krasovskii functional is much more challenging than its delay-free analogue, due to the fact that Lyapunov-Krasovskii functionals are defined on an infinitely dimensional function space. Another approach is based on Razumikhin-type results (see [9]), which relies on Lyapunov functions defined on Euclidean spaces. Roughly speaking, a Razumikhin theorem states that if the derivative of a Lyapunov function along trajectories is negative whenever the current value of the function dominates any value it takes over the interval of time lag, then the Lyapunov function will converge to zero along trajectories. There is an important link between the Razumikhin theorem and the small-gain theorem in the context of input-to-state stability, first recognized in [25]. The idea in [25] was developed further for control Lyapunov