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VISCOSITY APPROXIMATION OF FIXED POINTS FOR LIPSCHITZ PSEUDO-CONTRACTIONS [1]

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Abstract. The purpose of this paper is to prove strong convergence theorems for fixed points of pseudo-contractions in Banach spaces. The convergence of a continuous path and an explicit viscosity regularization iteration algorithms (VRIA) is provided in order to approximate fixed points of continuous or Lipschitz continuous pseudo-contractions. The results presented in this paper extend and improve those known results.

Keywords. Pseudo-contraction; Fixed point; Viscosity regularization iteration algorithm; Strong convergence.

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1 Introduction and Preliminaries

Throughout this paper, it is assumed that E is a real Banach space with norm $\|\cdot\|$ and let J be the normalized duality mapping from E into 2^{E^*} given by

$$Jx = \{x^* \in E^* : \langle x, x^*\rangle = \|x\|\|x^*\|, \|x\| = \|x^*\|\}$$

for all $x \in E$, where E^* denotes the dual space of E and $\langle\cdot,\cdot\rangle$ the generalized duality pairing between E and E^*. In the sequel, we denote a single-valued normalized duality mapping by j.

Recall that a mapping T with domain $D(T)$ and range in E is said to be pseudo-contractive if, for every $x, y \in D(T)$, there exists some $j(x-y) \in J(x-y)$ such that

$$\langle Tx - Ty, j(x-y)\rangle \leq \|x-y\|^2. \tag{1.1}$$

T is said to be strongly pseudo-contractive if there exists a constant $k \in (0,1)$ such that

$$\langle Tx - Ty, j(x-y)\rangle \leq k\|x-y\|^2, \tag{1.2}$$

for all $x, y \in D(T)$ and some $j(x-y) \in J(x-y)$.