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# ON THE DUAL VARIATIONAL METHOD FOR A SYSTEM OF NONLINEAR EQUATIONS WITH A PARAMETER

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**Abstract.** We use the dual variational method in order to prove that a certain system of difference equations with a parameter has at least two nontrivial bounded solutions which are obtained as the argument of a minimum and the argument of a maximum for the action functional associated with the problem under consideration.
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## 1 Introduction

In the recent years variational methods (or in other words critical point theory) have been extensively applied in showing the existence of solutions for certain types of two-point boundary value problems for difference equations arising mainly from evaluation of relevant differential equations (see [5] and references therein). In some cases the multiplicity of solutions is also considered. To mention a few papers we indicate [3], [4], [5], [6], [7] where mainly the mountain pass methodology is applied, the linking theorem is involved and the Palais-Smale type condition assumed. It seems that the variational approaches which are valid for the two-point boundary value problems can be applied for difference equations as well, however with some advances due to the simpler structure of the difference boundary value problem when compared with the differential BVP.

In order to provide some further insight into the problems - arising mainly from evaluation of boundary value problems for differential equations - we consider in this note the nonlinear system

$$Au = \lambda f(u), \ u \in R^N \tag{1}$$

with a positive parameter $\lambda$, a real positive definite symmetric $N \times N$ matrix $A$ and a continuous function $f(x) = [f_1(x_1), ..., f_N(x_N)]^T$, where $x_i \in R$; $T$ stands for transpose.