AN EXISTENCE THEOREM FOR THE COMMON SOLUTIONS FOR A PAIR OF INTEGRAL INCLUSIONS

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Abstract. In this paper we prove an existence theorem for the common solutions for a pair of integral inclusions via a common fixed point theorem of Turkoglu et al.[10].

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1 Introduction

Let $\mathcal{R}$ denote the real line, E be a Banach space with norm $\|.\|_E$ and let $C(E)$ denote the class of all nonempty closed subsets of E. Given a closed and bounded interval $J = [0,1]$ in $\mathcal{R}$, consider the integral inclusions

$$x(t) \in q(t) + \int_0^{\sigma(t)} k(t,s) F(s, x(s)) ds \qquad (1)$$

$$x(t) \in q(t) + \int_0^{\sigma(t)} k(t,s) G(s, x(s)) ds \qquad (2)$$

for $t \in J$, where $\sigma : J \to J$, $q : J \to E$, $k : J \times J \to \mathcal{R}$ are continuous and $F, G : J \times E \to C(E)$.

By a common solution for the integral inclusions (1) and (2), we mean a continuous function $x : J \to E$ such that

$$x(t) = q(t) + \int_0^{\sigma(t)} k(t,s) v_1(s) ds$$

and

$$x(t) = q(t) + \int_0^{\sigma(t)} k(t,s) v_2(s) ds$$