Dynamics of Continuous, Discrete and Impulsive Systems
Series **A:** *Mathematical Analysis* **18 (2011)** 41-52
 http://www.watam.org

GENERAL DECAY ESTIMATES FOR SECOND ORDER EVOLUTION EQUATIONS

Mounir Afilal[a] and Abdelaziz Soufyane[b]

[a]Département Mathématiques & Informatiques
Faculté Polydisciplinaire de Safi
Université Cadi Ayyad
Route Sidi Bouzid , B.P.: 4162-46000 Safi, Maroc.

[b]ALHOSN University, Faculty of Engineering & Applied Sciences
P.O.Box: 38772, Abu Dhabi, UAE

Corresponding author email: a.soufyane@alhosnu.ae

Abstract. This paper deals with a general decay estimates for a nonlinear second order integrodifferential. We establish a general decay result, from which the usual exponential and polynomial decay rates are only special cases. Our work allows certain relaxation functions which are not necessarily of exponential or polynomial decay and, therefore, generalizes and improves earlier results in the literature.

Keywords. Nonlinear systems; General decay; Multipliers techniques; Memory feedback; Relaxation function.

AMS (MOS) subject classification: 35G20, 93D15.

1 Introduction

In this paper we consider the following abstract integro-differential equation

$$u_{tt} + Au - \int_0^t \beta(t-s)Au(s)ds = \nabla F(u(t)), \quad t \in]0,\infty[\tag{1}$$

in a Hilbert space X, where $A : D(A) \subset X \to X$ is an accretive self-adjoint linear operator with dense domain, and ∇F denotes the gradient of a Gâteaux differentiable functional $F : D(\sqrt{A}) \to \mathbb{R}$.

This type of equations arise in viscoelasticity for example (see [7], [8]). The general decay of solutions for the viscoelasticity problems have attracted the attention of many mathematicians see for example ([9], [10]).

In [16], a class of abstract viscoelastic system of the form

$$\begin{cases} u_{tt} + Au + \delta u - \int_0^t g(t-s)A^\alpha u(s)ds = 0, \quad t \in]0,\infty[\\ u(0) = u_0, \quad u_t(0) = u_1. \end{cases}$$

for $0 \leq \alpha \leq 1$, $\delta \geq 0$, were investigated. When $0 < \alpha < 1$, Rivera et al. established that solutions for the abstract system decay polynomially even if g decay exponentially.