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EXISTENCE AND ITERATIVE APPROXIMATIONS OF BOUNDED NONOSCILLATORY SOLUTIONS OF HIGHER-ORDER NEUTRAL DELAY DIFFERENTIAL EQUATIONS

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Abstract. This paper deals with the following higher-order neutral delay differential equation with positive and negative coefficients:

$$[x(t) + cx(t-\tau)]^{(m)} + (-1)^m[P(t)x(f(t)) - Q(t)x(g(t))] = 0, \quad t \geq t_0,$$

where m is a positive integer, $c \in \mathbb{R}, \tau \in \mathbb{R}^+$, $P, Q \in C([t_0, +\infty), \mathbb{R}^+)$, $f, g \in C([t_0, +\infty), \mathbb{R})$ and $\lim_{t\to+\infty} f(t) = \lim_{t\to+\infty} g(t) = +\infty$. By using the Banach's fixed point theorem, we establish the existence of bounded nonoscillatory solutions for the above equation, construct some algorithms for approximating these bounded nonoscillatory solutions, and discuss the convergence and stability of iteration sequences generated by the algorithms. These results presented in this paper extend, improve and unify many known results due to Cheng and Annie [3], Kulenović and Hadžiomerspahić [8], Zhang and Yu [14], Zhou and Zhang [17] and others. Two examples are also included to dwell upon the importance of the results obtained in this paper.

Keywords. Higher-order neutral delay differential equation, bounded nonoscillatory solution, iteration scheme, stability, Banach's fixed point theorem.
AMS (MOS) subject classification: 34K15, 34C10.

1 Introduction

In this paper, we study the following higher-order neutral delay differential equation with positive and negative coefficients:

$$\begin{aligned}&[x(t) + cx(t-\tau)]^{(m)}\\ &\quad + (-1)^m[P(t)x(f(t)) - Q(t)x(g(t))] = 0, \quad t \geq t_0,\end{aligned} \tag{1.1}$$

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