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USE TIME SCALES TO STUDY IMPULSIVE FUNCTIONAL DIFFERENTIAL EQUATIONS

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Abstract. We explore connections between certain impulsive differential equations and corresponding dynamic equations on time scales. More specifically, for a given impulsive differential equation, we construct a "counterpart" as an equation defined on a time scale which has the same qualitative behavior. By doing so, we can simply "translate" any results for the equation on time scale to the impulsive differential equations. As applications of this result, we obtain oscillation criteria for two linear impulsive differential equations with delays by applying existing results for equations on time scales. Our work shows that this method provides a new approach for the general impulsive differential equations and can be used to study some equations which are difficult to deal with by the traditional ways.

Keywords. Dynamic equations on time scales, impulsive differential equations, equations with delays, oscillation, population model.

AMS (MOS) subject classification: 39A10, 34K45, 34K11

0 Introduction

Impulsive differential equations and dynamic equations on time scales are two different branches of differential equations. Both have theoretical importance and have been widely applied to many areas of science and engineering. In recent years, more and more mathematicians have been working on them and great progress has been made to each of them. In most work so far, these two types of equations are treated independently using different approaches.

In this paper, we explore the connections between impulsive differential equations and corresponding dynamic equations on time scales. More specifically, for a given impulsive differential equation, we construct a "counterpart" as an equation defined on a time scale which has the same qualitative behavior. By doing so, we can simply "translate" any results for the equation on time scale to the impulsive differential equations. This opens a new direction to study impulsive differential equations.

The idea is simple. Here we explain it with the first order ordinary dif-