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ON SOME SYSTEMS OF DIFFERENCE EQUATIONS WITH PERIODIC SOLUTIONS

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Abstract. This short note provides a simple proof of a generalization of some results given in B. D. Iričanin, and S. Stević, Some systems of nonlinear difference equations of higher order with periodic solutions. *Dyn. Contin. Discrete Impuls. Syst. Ser. A Math. Anal.* **13** (2006), no. 3-4, 499–507.

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1 Introduction

In [5], the authors consider systems of difference equations arising from interlacing well defined periodic solutions of some rational equations. Among similar results they prove that every positive solution of the system

$$x_n^{(1)} = \frac{1+x_{n-1}^{(2)}}{x_{n-2}^{(3)}}, \quad x_n^{(2)} = \frac{1+x_{n-1}^{(3)}}{x_{n-2}^{(4)}}, \quad \dots \quad , \quad x_n^{(k)} = \frac{1+x_{n-1}^{(1)}}{x_{n-2}^{(2)}}, \qquad n \geq 1, \quad (1)$$

where $k \in \mathbb{N}$ is fixed, is periodic with period equal to $5k$ if $k \neq 0 \mod 5$, and with period k if $k = 0 \mod 5$.

It is also shown in [5] that every positive solution of the system of equations

$$\begin{aligned} x_n^{(1)} &= \frac{1+x_{n-1}^{(2)}+x_{n-2}^{(3)}}{x_{n-3}^{(4)}}, \quad x_n^{(2)} = \frac{1+x_{n-1}^{(3)}+x_{n-2}^{(4)}}{x_{n-3}^{(5)}}, \quad \dots \quad , \\ x_n^{(k)} &= \frac{1+x_{n-1}^{(1)}+x_{n-2}^{(2)}}{x_{n-3}^{(3)}}, \qquad n \geq 1, \end{aligned} \quad (2)$$

is periodic with period $2^{3-i}k$ if $\gcd(k,8) = 2^i$, $i \in \{0,1,2,3\}$. These results generalize the well known periodicity of the associated scalar equations.

Here we give a generalization of these results thereby removing some of the complexities in [5].

For some further recent work on dynamics of solutions to systems of nonlinear difference equations see [1]–[7], and the references therein.