Dynamics of Continuous, Discrete and Impulsive Systems
Series **A:** *Mathematical Analysis* **15 (2008)** 871-884
 http://www.watam.org

FIXED POINT THEORY FOR VOLTERRA CONTRACTIVE OPERATORS OF MATKOWSKI TYPE IN FRÉCHET SPACES

Yeol Je Cho[1] and Donal O'Regan[2]

[1]Department of Mathematics Education and the RINS
College of Education
Gyeongsang National University
Chinju 660-701, Korea
E-mail: yjcho@gsnu.ac.kr
[2]Department of Mathematics
National University of Ireland
Galway, Ireland
E-mail: donal.oregan@nuigalway.ie

Abstract. We first prove a fixed point theorem for contractive maps of Matkowski type defined on a closed subset of a Fréchet space. Also we establish new Leray-Schauder results for contractive type maps between Fréchet spaces. The proof relies on fixed point theory in Banach spaces and viewing a Fréchet space as the projective limit of a sequence of Banach spaces.

Key Words: Fixed point theory, projective limits, contractive map of Matkowski type, Fréchet space, Leray-Schauder result.

AMS (MOS) Subject Classification: 1991 Mathematics Subject Classification: 47H10, 54H25.

1. Introduction

This paper presents new fixed point theorems for contractive type maps between Fréchet spaces. We begin by proving a fixed point theorem for contractive maps of Matkowski type defined on a closed subset of a Fréchet space. However in applications one would like to have a Leray-Schauder alternative in a Fréchet space setting. The usual Leray-Schauder alternatives in the non-normable situation are rarely of interest from an application viewpoint (this point seems to be overlooked by many authors) since the set constructed is usually open and bounded and so has empty interior. However it is possible to extend fixed point theory in Banach spaces to fixed point theory in Fréchet spaces and as a result in this paper we will obtain applicable Leray-Schauder alternatives in Fréchet spaces. In the literature [1, 2, 3, 6] one usually assumes the map F is defined on a subset X of a Fréchet space E and its restriction (again called F) is well defined on $\overline{X_n}$ (see Section 2). In general of course for Volterra operators the restriction is always defined on X_n and

[1]The corresponding author: yjcho@gsnu.ac.kr (Y. J. Cho).