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ITERATIVE SOLUTIONS FOR A CLASS OF NONLINEAR IMPULSIVE VOLTERRA INTEGRAL EQUATIONS IN BANACH SPACES

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Abstract. In this paper, by using the Banach fixed point theory, the existence and uniqueness theorem of iterative solutions for a class of nonlinear impulsive Volterra integral equations in Banach spaces is obtained in very weakly conditions. And then, this result is applied to infinite system of nonlinear impulsive Volterra integral equations. The results presented here improve and unify many known results.

Keywords. Banach Space, Nonlinear Operator, Impulsive Volterra Integral Equation, Banach Fixed Point Theory, Iterative Solution

AMS (MOS) subject classification: 34G20, 45N05

1 Introduction

Let E be a real Banach space and $\|\cdot\|$ denote the norm in E. Let $J = [0,a]$, $0 = t_0 < t_1 < \cdots < t_i < \cdots < t_p < a$ and $PC[J,E] = \{x \mid x$ is a map from J into E such that $x(t)$ is continuous at $t \neq t_i$, left continuous at $t = t_i$ and $x(t_i^+)$ exists for $i = 1,2,\cdots,p\}$. Evidently, $PC[J,E]$ is a Banach space with norm $\|x\|_{PC} = \sup_{t\in J}\|x(t)\|$.

Consider the nonlinear impulsive Volterra integral equations in E:

$$x(t) = x_0(t) + \int_0^t H(t,s,x(s))ds + \sum_{0<t_i<t} a_i(t)I_i(x(t_i)), \quad t \in J, \tag{1}$$

where $x_0 \in PC[J,E]$, $H \in C[D\times E,E]$, $D = \{(t,s) \in J\times J \mid t \geq s\}$, $I_i \in C[E,E]$ and $a_i \in C[J_i^*,R]$, $J_i^* = [t_i,a]$ $(i = 1,2,\cdots,p)$. $x \in PC[J,E]$ is called a solution of Eq.(1) if it satisfies (1) for all $t \in J$. In the following, let $a_i^* = \sup_{t\in J_i^*}|a_i(t)|$ $(i = 1,2,\cdots,p)$, $J_0 = [0,t_1]$, $J_1 = (t_1,t_2]$, $\cdots$, $J_{p-1} = (t_{p-1},t_p]$, $J_p = (t_p,a]$.

The theory of nonlinear impulsive Volterra integral equations in Banach spaces which is interested by many mathematical researchers has been emerging as an important area of investigation, because it has extensive applied background. It has arisen a series of results about this aspect, see [1–10]