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GLOBAL DYNAMICS OF UNBALANCED DELTA-MODULATED FEEDBACK-CONTROLLED DISCRETE-TIME SYSTEMS

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Abstract.[1] In this paper, a control policy called Unbalanced Δ-Modulated Feedback (UDMF) is proposed. For one-dimensional discrete-time systems with a parameter $0 < \mid a \mid \leq 1$, we show that a system of Type II has only two fixed points and the set of fixed points is globally attracting. Compared with systems of Type II, the evolutions of systems of Type I are much more complicated. For $0 < a < 1$, systems of Type I have no fixed points. Moreover, using a constructive method, we prove that there is a denumerable set of rate value $\gamma = \frac{\Delta_2}{\Delta_1}$. Corresponding to each parameter γ of the denumerable set, systems of Type I have no periodic orbits and, in this case, every orbit is dense in the state interval $[-\Delta_1,\ \Delta_2)$. To each of the other rate values of γ, systems of Type I all have an unique periodic orbit. In particular, the structural property of the periodic motion is robust; that is, there exists an interval including this value γ such that all parameters in this interval are corresponding to those periodic orbits of the same structural property. For the case of $a = 1$, all points in the interval $[-\Delta_1,\ \Delta_2)$ are $n-$periodic with $n \geq 3$ when γ is a rational number, and every orbit is dense in the interval $[-\Delta_1,\ \Delta_2)$ when γ is an irrational number. Moreover, every such unique periodic orbit is globally attracting for both types of systems.

Keywords. Unbalanced Δ-modulated feedback, Periodic orbit, Global attractor.

AMS (MOS) subject classification:

1 Introduction

As is well known, even a one-dimensional nonlinear system may have very complicated dynamics [2, 3, 6, 9].

In this paper, the following discrete-time nonlinear system is considered:

$$x_{n+1} = ax_n + u, \tag{1}$$

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