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STRONGLY NONLINEAR IMPULSIVE SYSTEM AND NECESSARY CONDITIONS OF OPTIMALITY[1]

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Abstract. In this paper, we consider optimal control of strongly nonlinear impulsive systems driven by impulse controls in infinite dimensions. PWC solution is introduced and existence of solutions for impulsive system is proved. We present the necessary conditions of optimality for a general Bolza problem. An example is also given.

Keywords. Optimal control; Nonlinear monotone operator; Impulsive system; Necessary Condition; Existence

AMS (MOS) subject classification: 34K35, 35D05, 93C24.

1 Introduction

Impulsive systems have been widely studied over many years because they arise in a lot of mathematical models of real world phenomena, such as the dynamics of populations subjected to abrupt changes (harvesting, diseases, etc.)([1], [12]). The impulsive systems on finite dimensional space have been studied in ([6], [8]). For the basic theory on impulsive differential equations, the reader is referred to Lakshmikantham([7]).

In recent years, impulsive evolution equations on infinite dimensional Banach space have been considered in several papers. Particularly, Ahmed considered optimal control of impulsive systems driven by impulsive controls in infinite dimensions ([2], [3], [7], and [9]). To our knowledge, although strongly nonlinear equations and optimal control are investigated by many authors including us ([4], [10], and [13]), only a few works (see [5], [11])have addressed strongly nonlinear impulsive control systems.

Let $I \equiv [0,T]$ be a closed bounded interval of the real line and define the set $D_1 \equiv \{t_1, t_2, \cdots, t_n\} \in (0,T)$. In this paper, we consider the following strongly nonlinear impulsive system driven by impulse controls

$$\left\{ \begin{array}{l} dx(t) + A(t,x(t))dt = g(t,x(t))dt + Q(\Upsilon x(t))dv, \quad t \in I \backslash D_1 \\ x(0) = x_0, \triangle x(t_i) = F_i(x(t_i)), i = 1,2,\cdots,n; \end{array} \right. \tag{1.1}$$

for $0 = t_0 < t_1 < \cdots < t_n < T$, where $A(\cdot,\cdot)$ is a nonlinear monotone operator, g is an nonlinear nonmonotone perturbation, $v \in BV(I,U)$, and

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