Dynamics of Continuous, Discrete and Impulsive Systems
Series **A:** *Mathematical Analysis* **12 (2005)** 457-468

EXISTENCE AND GLOBAL ATTRACTIVITY OF POSITIVE PERIODIC SOLUTION FOR AN IMPULSIVE DELAY POPULATION MODEL

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Abstract. In this paper we shall consider the following nonlinear impulsive delay population model

$$\begin{cases} x'(t) + p(t)x(t) - \frac{q(t)x(t)}{r+x^n(t-m\omega)} = 0, \text{a.e. } t > 0, t \neq t_k, \\ x(t_k^+) = (1+b_k)\,x(t_k), k = 1, 2, \cdots, \end{cases}$$

where $m \geq 0$ and $n > 0$ are integers, $p(t)$ and $q(t)$ are positive periodic continuous functions with period $\omega > 0$, and $r > 0$ is a constant. In the nondelay case ($m = 0$) we shall show that the above system has a unique positive periodic solution $x^*(t)$, and provide sufficient conditions for the global attractivity of $x^*(t)$. In the delay case we shall present sufficient conditions for the persistence of the above system, and establish sufficient conditions for the global attractivity of $x^*(t)$. It is shown that under the appropriate linear periodic impulsive perturbations, the impulsive delay equation preserves the original periodicity properties of the nonimpulsive delay equation, but the global attractivity of it depends on variation of the coefficient b_k.

Keywords. Existence; global attractivity; positive periodic solution; impulsive; delay differential equation.

AMS (MOS) subject classification: 34D20, 92D25.

1 Introduction

Many differential equations and delay differential equations have been proposed to model the variation of biological and ecological dynamical systems. In which the parameters' characteristics reflect the outer environment imposed upon the systems. Recently, there is a growing interest in the study of periodic population dynamics, just as Cushing [3] pointed out that it is necessary and important to consider models with periodic ecological parameters or perturbations which might be quite naturally exposed, for example, those due to seasonal effects of weather, food supply, mating habits, hunting or harvesting seasons, etc., several such kinds of models have been investigated in [4, 9, 19, 20]. Usually these perturbations are treated continually, while there are still other perturbations such as fires, floods, etc. that are not suitable to be considered continually. Many evolution processes in nature are characterized by the fact that at certain moments of time there