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A NEW APPROACH TO ELLIPTIC EQUATIONS USING A LERAY–SCHAUDER ALTERNATIVE FOR A SUBCLASS OF WEAKLY SEQUENTIALLY CONTINUOUS MAPS†

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Abstract. An existence principle is presented for elliptic Dirichlet problems using a new Leray–Schauder alternative for weakly–strongly sequentially continuous maps.

1 Introduction

In this paper we present an existence principle for the elliptic Dirichlet problem

$$\begin{cases} \triangle y + f(t,y) = 0 \quad \text{on} \quad \Omega \\ y = 0 \quad \text{on} \quad \partial\Omega; \end{cases} \tag{1.1}$$

here Ω is a bounded domain in $\mathbf{R}^n$, $n \geq 3$, with a $C^{1,1}$ boundary $\partial\Omega$. In this paper we are interested in strong solutions to (1.1) and our results extend and complement those in [4, 6]. Our theory is based on a new Leray–Schauder alternative for weakly–strongly sequentially continuous maps. This alternative combines the advantages of the strong topology (i.e. the sets will be open in the strong topology) with the advantages of the weak topology (i.e. the maps will be weakly–strongly sequentially continuous and weakly compact) and we will see in Section 2 how easily our alternative applies to (1.1). One of the disadvantages of the standard Leray–Schauder alternative in the literature [2] is that a lot of work is usually spent checking the compactness of the map. However if one uses this new approach the weak compactness of the map is immediate.

For notational purposes [1, 5] for a nonnegative integer k and a real number $p \in \left(\frac{n}{2}, \infty\right)$ we denote by $W^{k,p}(\Omega)$ the space of all real valued functions defined on Ω whose weak partial derivatives up to order k lie in $L^p(\Omega)$, equipped with the usual norm. $W_0^{1,p}(\Omega)$ stands for the closure of

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