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BILINEAR CONTROLLABILITY PROPERTIES OF A VIBRATING STRING WITH VARIABLE AXIAL LOAD AND DAMPING GAIN.

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Abstract We show that the set of equilibrium states like $(y_d, 0)$ of a vibrating string that can approximately be reached in $H_0^1(0,1) \times L^2(0,1)$ by varying its axial load and the gain of damping is dense in the subspace $H_0^1(0,1) \times \{0\}$ of this space. Our argument is constructive

Keywords. wave equation, approximate controllability, bilinear control, axial load, damping.

1. Introduction

1.1. Problem setting. In this paper we consider the following initial and boundary-value problem for the one dimensional wave equation modeling oscillations of a damped vibrating string with clapped ends and an axial load:

$$y_{tt} = y_{xx} + v(x,t)y - \gamma(t)y_t, \quad x \in (0,1), \ \ t > 0, \tag{S}$$
$$y(0,t) = y(1,t) = 0, \quad y(x,0) = y_0(x), \ \ y_t(x,0) = y_1(x).$$

In the above $v(x,t)$ is the axial load at point x and time t and $\gamma(t)$ is the gain of the viscous (motion-activated) damping acting upon the string at time t.

Our goal is to investigate the controllability properties of model (S), assuming that one can vary the values of axial load v and of the gain γ, which we further regard as bilinear (multiplicative) controls. Namely, *given the (non-zero) initial state* (y_0, y_1)*, we would like to know what states* $(y(\cdot,t), y_t(\cdot,t))$ *can be achieved by system (S) at times* $t > 0$ *by applying various aforementioned bilinear controls* v *and* γ.

A problem like this also arises in a more general context of so-called "smart materials", whose properties can be altered by applying various external factors, such as temperature, electrical current, or magnetic field.

Remark 1.1. One can easily notice that the zero state $(y_0, y_1) = (0,0)$ is the fixed point for the solution mappings of system (S), regardless of the choice of controls v and γ. Hence, it cannot be steered anywhere from this