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EXPONENTIAL STABILITY FOR A CLASS OF LINEAR TIME-VARYING SINGULARLY PERTURBED STOCHASTIC SYSTEMS

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Abstract. In this paper the problem of exponential stability for a class of singularly perturbed systems of stochastic differential equations with multiplicative white noise is investigated. It is shown that if the zero solutions of two subsystems independent of the small parameter are exponentially stable, then the zero solution of the given full system is exponentially stable too, and some estimates of the block components of the fundamental matrix solutions are obtained. Conversely, if such estimates of the block components of the fundamental matrix solution of the full system are fulfilled, then the zero solutions of the so called boundary layer subsystem and of some reduced subsystem are exponentially stable. One of the conclusion of this paper is that for singularly perturbed systems of Ito differential equations, the corresponding reduced subsystem cannot be associated, via simply neglecting the small parameters, as in the deterministic framework.
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1 Introduction

The systems of differential equations with singular perturbations were intensively investigated in the last fifty years starting with the pioneered work of Tichonov [13]. We recall that a singularly perturbed system of differential equations contains small parameters as coefficients of the derivatives of some unknown functions of the system. Usually such small parameters are neglected, thus we may associate two subsystems of lower dimensions which are independent of the small parameters, namely the fast subsystem or the boundary layer subsystem and the reduced subsystem or the slow subsystem. The problem is to show under what conditions the solutions of the obtained subsystems provide a good approximation of the solutions of the given system. Another problem is to show how the stability of some solutions of the reduced subsystem and of the boundary layer subsystem, respectively, provide stability of some solution of the given full system.

In the work [10] it was proved that the exponential stability of the zero solution of the reduced subsystem and of the zero solution of the boundary