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COLLISION OF UNIFORM SOLITON TRAINS IN ASYMMETRIC SYSTEMS

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Abstract. This paper describes the original discovery of soliton collisions governed by the forced Korteweg-de Vries equation and forced nonlinear Schrödinger equation, respectively. These two nonlinear dynamic systems do not have infinitely many conservation laws and are grouply asymmetric due to external forcing. The forcing makes it possible to generate a train of solitary waves of the same size. The traditional group-theoretical method is no longer appropriate for describing these solitary waves. This paper numerically demonstrates that the collision process of the solitary waves generated by the forcing in the two asymmetric dynamic systems, hence confirms that the solitary waves are solitons.

Keywords. Forced solitons, soliton collision, forced Korteweg-de Vries equation, forced nonlinear Schrödinger equation, numerical method.
AMS (MOS) subject classification: 35Q51, 76C10

1 Introduction

In the milestone paper on modern nonlinear science, Zabusky and Kruskal (1965) studied Fermi, Pasta and Ulam's discovery of recurrence of energy distribution and showed that certain traveling waves can pass each other and retain their original shapes [8]. These waves are called "solitons" (i.e., lonely particles) and they have produced a historically stronger than ever stimulation of the research into the nonlinear-wave mathematics with symmetries. These symmetries are equivalent to the existence of infinitely many conservation laws. Various beautiful mathematics has been generated from these symmetries, such as soliton-hierarchy in Lie algebra, shape transition in geometry, Backlund transform, and inverse-scattering method. Unfortunately, when certain symmetries, such as the translation invariant property, are broken, or certain conservation laws, such as the conservation of momentum, are not satisfied, the above mathematics is no longer working. In the practical world, asymmetry is common and absolute symmetry is rare. An outstanding question is that: are there still solitons, under the collision definition of Zabusky and Kruskal, governed by asymmetric dynamic systems?